Y.V.N.R Govt. Degree College, Kaikaluru, Eluru (D).

I B.Sc MATHEMATICS.

SEMESTER-I PAPER I : DIFFEENTIAL EQUATIONS

QUESTION BANK

Unit-I : FIRST ORDER AND FIRST DEGREE DIFFERENTIAL EQUATIONS

4Marks :

1. Solve $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$
2. Solve $\frac{dy}{dx} + 2xy = e^{-x^2}$.
3. Solve $x\frac{dy}{dx} + 2y = x^2 \log x$.
4. Solve $xy\,dx - (x^2 + 2y^2)dy = 0$
5. Solve $\frac{dy}{dx} + (2x\tan^{-1}y - x^3)(1 + y^2) = 0$.
6. Solve $(1+e^{\frac{x}{y}})\,dx+(1-\frac{x}{y})\,e^{\frac{x}{y}}\,dy=0$.
7. Solve $(y^4+2y)\,dx+(xy^3+2y^4-4x)\,dy=0$.

8 Marks:

8. Solve $x\frac{dy}{dx} + y = y^2 \log x$.
9. Solve $x(x-1)\frac{dy}{dx} - y = x^2(x-1)^2$.
10. Solve $\frac{dy}{dx}(x^2y^3+xy) = 1$.
11. Solve $x^2y\,dx - (x^3+y^3)\,dy=0$.
12. Solve $y(1+xy)\,dx + x(1-xy)\,dy=0$.
13. Solve $2xy\,dy-(x^2+y^2+1)\,dx=0$.

Unit-II : ORTHOGONAL TRAJECTORIES

4Marks :

1. Find the orthogonal trajectories of the family of hypocycloids $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ where 'a 'is a parameter.
2. Find the orthogonal trajectories of the family of curves $r = a\theta$, where 'a 'is a parameter.

3. Solve $x^2(\frac{dy}{dx})^2 + xy\,\frac{dy}{dx} - 6y^2 = 0$.

4. Solve $p^2x^2 = y^2$.

5. Solve $y = 2px - p^2$.

6. Solve (y-xp)(p-1)=p by using clairauts form.

8 Marks :

7. Show that the family of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self orthogonal ,where λ is parameter .

8. Show that the system of confocal and coaxial parabolas $y^2 = 4a\,(x + a)$ is self orthoganal where a is a parameter.

9. Find the orthogonal trajectory equations of $r = \frac{2a}{1+cos\theta}$,where a is a parameter .

10. Solve $p^2 + 2pycosx = y^2$.

11. Solve $y = 2px + x^2p^4$.

12. Solve $y^2\, logy = xyp + p^2$.

Unit-III : HIGHER ORDER LINEAR DIFFERENTIAL EQUATIONS

4Marks :

1. Solve $(D^2 + 4D + 3)y = e^{2x}$.
2. Solve $(D^3 - 5D^2 + 8D - 4)y = e^{2x}$.
3. Solve $(D^2 - 5D + 6)y = e^{x}$.
4. Solve $(D^2 - 1)y = cosx$.
5. Solve $(D^2 + 3D + 2)y = e^{-2x} + sinx$.

8 Marks:

6. Solve $(D^2 - 3D + 2)y = coshx$.
7. Solve $(D^3 - 5D^2 + 7D - 3)y = e^{2x}coshx$.
8. Solve $(D^3 + 1)y = 3 + 5e^{2x}$.
9. Solve $(D^2 - 4D + 3)y = sin3x\, cos2x$.
10. Solve $(D^2 - 3D + 2)y = cos3x\, cos2x$.
11. Solve $(D^2 + 4)y = e^{x} + sin2x + cos2x$.

Unit-IV : HIGHER ORDER LINEAR DIFFERENTIAL EQUATIONS

4 Marks :

1. Solve $(D^2 + D + 1)y = x^3$.
2. Solve $(D^2 - 4D + 4)y = xe^{2x}$.
3. Solve $(D^2 - 6D + 13)y = 8e^{3x} sin2x$.
4. Solve $(D^2 - 7D + 6)y = e^{2x}(1 + x)$.
5. Solve $(D^3 + 2D^2 + D)y = x^2 + x$.
6. Solve $(D^2 - 1)y = xsinx$.

10 Marks:

7. Solve $(D^2 + 3D + 2)y = e^{-x} + x^2 + cosx$
8. Solve $(D^2 - 2D + 4)y = 8(x^2 + e^{2x} + sin2x)$.
9. Solve $(D^2 - 4D + 4)y = 8x^2 e^{2x} sin2x$.
10. Solve $(D^2 + 4)y = xsinx$.
11. Solve $(D^2 + 2D + 1)y = xcosx$.
12. Solve $(D^2 + 1)y = x^2 sin2x$.

Unit-V :HIGHER ORDER LINEAR DIFFERENTIAL EQUATIONS

4 Marks :

1. Solve $(x^2 D^2 - 2xD - 4)y = x^2$.
2. Solve $(x^2 D^2 + xD - 4)y = x^2$.
3. Solve $(x^2 D^2 - xD + 1)y = logx$.

10 Marks:

4. Solve $(D^2 + 1)y = cosecx$ by the method of variation of parameters.
5. Solve $(D^2 + a^2)y = tan\ ax$ by the method of variation of parameters.
6. Solve $(D^2 + 4)y = 4\ sec^2 2x$ by the method of variation of parameters.
7. Solve $(x^2 D^2 - xD + 2)y = x\ logx$.
8. Solve $(x^2 D^2 - xD + 4)y = \cos(logx) + x \sin(logx)$.
9. Solve $(x^2 D^2 + 2xD - 20)y = (x + 1)^2$.

Y.V.N.R Govt. Degree College, Kaikaluru, Eluru (D).

II B.Sc MATHEMATICS.

SEMESTER-III

PAPER III : GROUP THEORY

IMPORTANT QUESTIONS

GROUPS

4Marks :

1. In a group G show that $(a\,b)^{-1} = b^{-1}\,a^{-1}$ for all a ,b ∈G
2. Prove that every group has unique identity.
3. Prove that In a group G each element has unique inverse.
4. Show that a group G is abelian iff $(a\,b)^2 = a^2\,b^2$ for all a ,b ∈ G
5. If G is a group such that $a^{-1} = a$ for all a ∈G then prove that G is abelian.
6. Show that the set of 4^{th} roots of unity G ={ *1,-1, i, -i* } *form* a groupwrt multiplication.

8 Marks:

7. If G is a Group and a ,b ∈ G then prove that a x = b and y a=b haveunique solutions.
8. Show that G = { $a + b\sqrt{2}$ / a ,b ∈ Q } is a group with respect to multiplication.
9. Show that G = $\left\{\begin{pmatrix} \cos a & -sina \\ sina & cosa \end{pmatrix} / \alpha \in Z\right\}$ is a group w r t multiplication of matrices.
10. Show that the set of Positive Rational numbers Q^+ is an abelian group w. r .t an operation * defined by $a*b = \frac{ab}{3}$ for all a,b ∈ Q^+.
11. Show that n^{th} roots of unity form an abelian group under multiplication ofComplex numbers .

Sub groups

4 Marks :

1. Show that the intersection of two subgroups is a subgroup.
2. Let G be a group and a ∈ G . If H ={ x ∈ G/ a x = x a } then show that H is a subgroup of G.
3. Show that if H is a subgroup of Group G iff $HH^{-1} = H$.
4. If H is any subgroup of a group G then $H^{-1} = H$.
5. Let H is a subgroup of a group G then prove that Ha = Hb ⇔ ab^{-1} ∈ H.
6. Let H is a subgroup of a group G then prove that a ∈ Hb ⇔ Ha = Hb.

8 Marks :

7. A non-empty subset H of a group G is a subgroup of G iff for all a ,b $\in$H $\Rightarrow$ $\mathbf{ab^{-1}}$ $\in$H.
8. Let H and K are two subgroups of a group G then prove that HK is a subgroup If and only if HK=KH.
9. Let H_1 and H_2 are subgroups of a group G then prove that $H_1 \cup H_2$ is a subgroup iff $H_1 \subseteq H_2$ or $H_2 \subseteq H_1$
10. Prove that any two right or left cosets are either identical or disjoint.
11. State and prove Lagrange's theorem for finite groups.

NORMAL SUBGROUPS

4 Marks:

1. Prove that a subgroup H of a group is normal $\Leftrightarrow$ xHx^{-1} =H for all x $\in$ G.
2. Show that every subgroup of index 2 is normal subgroup.
3. Show that Intersection of two normal subgroups is normal subgroup.
4. Prove that Every subgroup of an abelian group is normal .

8 Marks:

5. Prove that a subgroup H of a group G is normal if and only if each left coset of H in G is a right coset of H in G .
6. Prove that a subgroup H of a group G is normal iff product of two right coset of H in G is again a right coset of H in G.
7. If M and N are two normal subgroups of a group such that $M \cap N = \{e\}$ then each element in M commute with each element in N.
8. If H is a normal subgroup of a group G then prove that G/H = { H x / x $\in$ G } is a group w r t coset multiplication

Homomorphism s and permutations

4Marks :

1 If f: $G \to G'$ is a homomorphism then prove that kernel of f is a normal subgroup of the group G.

2. If f: $G \to G'$ is a homomorphism defined by $f(x) = x^2$ for all $x \in G$ then show that G is abelian .
3. If f: $G \to G'$ is a homomorphism defined by $f(x) = x^{-1}$ for all $x \in G$ then show that G is abelian.
4. If $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}$ write into product of disjoint cycles .

5. If $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 4 & 5 & 6 & 7 & 1 \end{pmatrix}$ is odd permutation .

6. If G = { 1, ω, ω^2 } is a group then find all the regular permutations of G .

8 Marks :

7 State and prove Fundamental theorem of homeomorphisms'.

8 If f: G⇢G' is a homomorphism then prove that f is isomorphism iff ker f={e}.

9. Let G be a Group and f: G⇢G such that f(a) = a^{-1} for a ∈ G prove that f is isomorphism if and only if G is commutative.

10. State and prove cayley's theorem for permutation groups.

11. If f= (1 2 3 4 5 8 7 6) g= (4 1 5 6 7 3 2 8) then show that $(f g)^{-1} = g^{-1} f^{-1}$.

12. State cayley's theorem for permutation groups. If G = {***1,-1, i, -i*** } is a group then find all the regular permutations of G .

RINGS

4 Marks :

1.Prove that every field is an integral domain.

2.Prove that in a Boolean ring R i) a + a = 0 OCT-17 ii) if a + b = 0 then a= b iii) R is a commutative ring

3. Show that the characterstic of a boolean ring is 2.

8 Marks :

4. Prove that every finite integral domain is a field
5. Show that the set of Gaussian integers Z(*i*) ={ a + *i* b / a, b Ɛ Z } form an integral domain. Is it a field ? justify your answer.
6. Show that the set Q (√ 2) = { a + b√ 2 / a, b Ɛ Q } is a field with respect to addition and multiplication .
7. Prove that the characterstic of an integral domain is either zero or prime.

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II B.Sc Semester - V

SUBJECT:MATHEMATICS

PAPER V - LINEAR ALGEBRA & MATRICES

Question Bank

UNIT-I:VECTOR SPACES – I (10 M)

1. Prove that the necessary and sufficient condition for a non-empty sub set W of a vector space V(F) to be subspace of V is that $a,b \in F, \boldsymbol{\alpha}, \boldsymbol{\beta} \in \mathbf{W} \Rightarrow a\boldsymbol{\alpha} + \mathbf{b}\boldsymbol{\beta} \in \mathbf{W}$.
2. Let W_1 and W_2 be two Subspaces of a vector space V(F) then Prove that $W_1 \cup W_2$ is a subspace of V(F) iff $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$.
3. If S is a non-empty subset of a vector space V(F) then prove that $L(S) = < S >$
4. If $\alpha_1, \alpha_2, \alpha_3 \dots \dots \dots . \alpha_n$ are non zero vecti=or in a vector space in V(F) then prove that either they are linearly independent or one vector α_k where $2 \leq k \leq n$ is a linear combination of preceding vectors $\alpha_1, \alpha_2, \alpha_3 \dots \dots \dots . \alpha_{k-1}$

VECTOR SPACES –I (4 M)

5. If W_1 and W_2 are two sub spaces of vector space V(F) then Prove that $W_1 \cap W_2$ is also Subspace of V(F).

6. If S and T are two subsets of the vector space V(F) then show that $L(S \cup T) = L(S) + L(T)$.

7. Express the vector $\boldsymbol{\alpha} = (1,-2,5)$ as a linear combination of the vectors $\boldsymbol{\alpha}_1 = (1,1,1)$ $\boldsymbol{\alpha}_2 = (1,2,3)$ $\boldsymbol{\alpha}_3 = (2,-1,1)$ in $V_3(F)$.

8. Show that the vectors (1,3,2) (1,-7,-8)(2,1,-1) of $V_3(R)$ are linearly dependent.

9. Show that the vectors (1,2,1) (3,1,5)(3,-4,7) of $V_3(R)$ are linearly dependent .

10. Show that the vectors (1,2,0) (0,3,1)(-1,0, 1) of $V_3(R)$ are linearly independent.

11. Show that the vectors (1,0,1) (1,1,0)(1,-1,1) of $V_3(R)$ are linearly independent .

UNIT-II:VECTOR SPACES –II (10 M)

1. Prove that every finite dimensional vector space has a basis.

2. If V(F) is finite dimensional Vector space then prove that any two basis of V have the same number of elements.

3. Prove that every linearly independent sub set of a finite dimensional vector space V(F) is either basis of V or can be extended to form a basis of V.

4. Show that the vectors{ (2,1,0),(2,1,1),(2,2,1) } form a basis of V_3 (R) and express the vector (1,2,1) as a linear combination of these basis vectors

5. If W_1 and W_2 are two sub spaces of a finite dimensional vector space V(F) then prove that $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

6. If W is subspace of a finite dimensional vector space V(F) then Prove that $\dim(V/W) = \dim V - \dim W$.

VECTOR SPACES –II (4 M)

7. Show that the vectors (1,0,0),(0,1,0)(0,0,1) is a basis of the vectpr space $v_3(F)$
8. Show that the vectors (1,2,1),(2,1,0)(1,-1,2) form a basis for R^3.
9. Show that the vectors (1,1,2),(1,2,5)(5,3,4) not a basis for R^3.
10. If $\{\alpha, \beta, \gamma\}$ is a basis for $V_3(R)$ then show that $\{\alpha + \beta, \beta + \gamma, \gamma + \alpha\}$ is also a *Basis of* $V_3(R)$.

UNIT-III:LINEAR TRANSFORMATIONS (10 M)

1. State and prove Rank – Nullity theorem.

2. Show that the mapping $T: R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x+2y-z, y + z, x+ y-2z)$ is a linear transformation .Find its rank , nullity and verify rank T + nullity T $= \dim R^3$.

3. Verify Rank – Nullity theorem for a linear Transformation $T: R^3 \rightarrow R^3$ defined by $T(x, y, z)=(x-y, 2y+z, x+y+z)$ and also Show that T ia s Linear Transformation.

LINEAR TRANSFORMATIONS (4 M)

4. T is a linear transformation from a vector space U(F) into a vector space V(F) then Prove that range of T is R(T) is a subspace of V(F).

5. T is a linear transformation from a vector space U(F) into a vector space V(F) thenProve that the null space of T is N(T) is a subspace of U(F).

6. Show that the mapping $T: V_2(R) \rightarrow V_3(R)$ is defined by $T(a, b)= (a + b, a- b, b)$ ia a linear transformation from $V_2(R)$ in to $V_3(R)$.

UNIT-IV:MATRICES (10 M)

1. Solve the system $\lambda x + y + z = 0, x + \lambda y +z = 0, x + y + \lambda z = 0$ if the system of equations has non zero solution.

2. Solve $4x + 2y + z + 3u =0, 6x + 3y + 4z + 7u =0, 2x + y + u =0$ the system of equations Completely.

3. Show that the system of equations $x + y + z = 6, x + 2y + 3z = 14, x + 4y + 7z =30$ are Consistent and hence solve them.

4. For what values of λ, the equations $x + y + z = 1, x +2y + 4z = \lambda, x+4y+10z = \lambda^2$ have solution? Solve them completely in each case.

5. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

6. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

7. State and prove Cayley - Hamilton theorem.

8. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ verify that it is satisfied by A and hence find A^{-1}

9. Verify Cayley-Hamilton thorem for the matrix A = $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ hence find A^{-1}

MATRICES (4 M)

10. Solve Completely the system of equations $x + 3y - 2z = 0, 2x - y + 4z = 0, x - 11y + 14z = 0$

11. Show that the system of Equations $x + y + z = -3, 3x + y - 2z = -2, 2x + 4y + 7z = 7$ *are inconsistent.*

12. Show that the system of Equations $2x - y + 3z = 8, -x + 2y + z = 4, 3x + y - 4z = 0$ *are Consistent.*

13. Find Eigen Values of Matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$

14. Find Eigen Values of Matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -2 & 0 \end{bmatrix}$

15. Verify Characteristic equation of a Matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

16. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ *find Inverse of A by Using Cayley – Hamilton theorem*

UNIT-V: INNER PRODUCT SPACES (10 M)

1. State and prove Schwarz's Inequality.
2. If α, β are two vectors in an inner product space V(F) then prove that $|(\alpha, \beta)| = \|\alpha\| \, \|\beta\|$ if and only if α, β are linearly dependent.
3. If α, β are two vectors in a unitary space V(C) then Prove that $4(\alpha, \beta) = \|\alpha + \beta\|^2 - \|\alpha - \beta\|^2 + i\|\alpha + i\beta\|^2 - i\|\alpha - i\beta\|^2$

4. State and prove Bessel's inequality.Type equation here.
5. Apply the Gram-Schmidt's process to the vectors { (2,1,3),(1,2,3) (1,1,1) } to obtain an orthonormal basis for $V_3(R)$ with the standard inner product.

6. Apply the Gram-Schmidt's process to the vectors { (1,0,1),(1 ,0,-1) (0,3,4) } to obtain an orthonormal basis for $V_3(R)$ with the standard inner product.

INNER PRODUCT SPACES(4 M)

7. State and prove Triangle inequality.

8. State and prove Parallelogram Law.

9. If α, β are two vectors in an inner product space V(F) and a, b$\in$ F , $\alpha, \beta \in V(F)$ then show that $\text{Re}(\alpha, \beta) = \frac{1}{4} \| \alpha + \beta \|^2 - \frac{1}{4} \| \alpha - \beta \|^2$

10. If α, β are two vectors in an inner product space V(F) such that $\| \alpha \| = \| \beta \|$ then Prove that $\alpha + \beta$, $\alpha - \beta$ are orthogonal.

11. Prove that every orthogonal set of non zero vectors in an inner product space V(F) is linearly independent.

12. Prove that every orthonormal set of non zero vectors in an inner product space V(F) is linearly independent

Y.V.N.R Govt. Degree College, Kaikaluru, Eluru (D).

III B.Sc Semester - V

SUBJECT:MATHEMATICS

PAPER 6A – NUMERICAL METHODS

Question Bank

UNIT – I: FINITE DIFFERENCES AND INTERPOLATION WITH EQUAL INTERVALS:

10 Marks:

1. State and prove Fundamental theorem of finite differences .
2. Estimate the missing terms in the following data

x	0	1	2	3	4	5
f(x	0	-	8	15	-	35

3.State and prove Newton's forward interplation formula .

4.Using Newton's forward interplation formula find the value of $sin52^0$.

x^0	45	50	55	60
$sinx^0$	0.7071	0.7660	0.8192	0.8660

5. State and prove Newton's backward interplation formula .

6. Using Newton's backward interplation formula find the value of $tan17^0$.

x	4	8	12	16	20	24
f(x)	0.0699	0.1405	0.2126	0.2867	0.3640	0.4452

4 Marks:

1. State and prove Advancing difference formula .

2. Show that $e^x = \frac{\Delta^2}{E} e^x \frac{Ee^x}{\Delta^2 e^x}$ where h=1 .

3.Show that $\sqrt{1+\delta^2\mu^2} = 1 + \frac{1}{2}\delta^2$.

4. Estimate the missing terms in the following data:

X	1	2	3	4	5	6	7
f(x)	2	4	8	-	32	64	128

5. Find the factorial polynomial of $11x^4 + 5x^3 + 2x^2 + x - 15$.

6. Find the factorial polynomial of $x^4 - 12x^3 + 24x^2 - 30x + 9$.

UNIT II: INTERPOLATION WITH EQUAL AND UNEQUAL INTERVALS:

10 Marks :

1. State and prove Gauss's forward central difference formula .
2. State and prove Gauss's backward central difference formula .
3. Find the value of log 37 using Gauss's forward interpolation formula

x	25	30	35	40	45
f(x)	1.3979	1.4771	1.5440	1.6020	1.6532

4. State and prove Newton's divided difference formula.

5. State and prove Lagranges divided difference formula.

6. By means of Newton's divided difference formula find the value of f(8) and f(15)from the following table

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

4 Marks :

1. Find by Gauss's backward formula the sale of a concern for the year 1946

Years(x)	1931	1941	1951	1961	1971
Sales in (x)	15	20	27	39	52

2.State and prove Stirling's formula.

3.State and prove Bessel's formula .

4.If $f(x) = \frac{1}{x}$ then find f (a,b) and f (a,b,c) .

5.By Lagrange's interpolation formula find the value of f(5) given that

x	1	2	4	8	10
f(x)	8	15	19	32	40

6. By Lagrange's interpolation formula find the value of y at x=5, given that

x	1	3	4	8	10
y	8	15	19	32	40

UNIT III :NUMERICAL DIFFERENTIATION:

10 Marks :

1. Using the following table compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1 .

x	1	2	3	4	5	6
y	1	8	27	64	125	216

2.Using the given table find $\frac{dy}{dx}$ *and* $\frac{d^2y}{dx^2}$ at x = 2.2 .

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

3. Using newton's divided difference formula find the value of $f'(4)$ and $f'(5)$.

x	1	2	4	8	10
f(x)	0	1	5	21	27

4.Using the given table find $\frac{dy}{dx}$ *and* $\frac{d^2y}{dx^2}$ at x = 1.6 .

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
f(x)	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

4 Marks:

1. Using the following table compute $\frac{dy}{dx}$ at x = 1 .

x	1	2	3	4	5	6
y	1	8	27	64	125	216

2.Using the following table compute $\frac{dy}{dx}$ at x = 6 .

x	1	2	3	4	5	6
y	1	8	27	64	125	216

3.From the following table,find the value of x for which y is minimum and find this value of y .

x	0.60	0.65	0.70	0.75
f(x)	0.6221	0.6155	0.6138	0.6170

4.From the following table find x correct to two decimal places ,for which y is maximum and find this value of y .

x	1.2	1.3	1.4	1.5	1.6
y	0.9320	0.9636	0.9855	0.9975	0.9996

UNIT IV: <u>NUMERICAL INTEGRATION:</u>

10Marks :

1.Evaluate $\int_4^{5.2} log_e x dx$ by using Trapezodial rule .

2. Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ by using Simpson's 3/8 rule.

3. Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ by using Simpson's 1/3 rule .

4.Calculate an approximate value of $\int_0^{\pi/2} sinx\, dx$ using 11 ordinate by using Trapezodial rule .

5.Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ by using Simpson's 3/8 rule .

6. Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ by using Weddle's rule.

4 Marks :

1.Evaluate $\int_{-3}^{3} x^4 dx$ by Trapezodial rule with h = 1.

2.Evaluate $\int_0^1 \frac{1}{1+x} dx$ by using Simpson's 3/8 rule .

3. Evaluate $\int_0^7 \frac{1}{x} dx$ by using Simpson's 1/3 rule .

4. Evaluate the integral $\int_4^{5.2} logx\ dx$ by using Weddle's rule .

5. Evaluate $\int_0^{\pi/2} sinx\ dx$ by using Euler-Mclaurin's formula .

6.Using Euler-Mclaurin's formula with n = 4 to estimate $\int_0^1 \frac{1}{1+x^2} dx$.

UNIT V :NUMERICAL SOLUTION OF ORDINAY DIFFERENTIAL EQUATIONS:

10 Marks :

1.Using the Tylor's series method for $y(x)$, find $y(0.1)$ correct to four decimal places if $y(x)$ satisfies $y' = x - y^2, y_0 = 1, where\ x_0 = 0$.

2.Solve $y' = y - x^2, y(0) = 1$ by Picard's method up to the fourth approximation.Hence find the value of $y(0.1), y(0.2)$.

3.Given $y' = \frac{y-x}{y+x}$ with $y_0 = 1$,find $y\ for\ x = 0.1$ in 4 steps by Euler's method .

4.Solve the equation $y' = x + y$ with $y_0 = 1$ by Runge-Kutta rule from $x = 0\ to\ x = 0.4\ with\ h = 0.1$.

4Marks :

1. Using Tylor's series method ,solve the equation $\frac{dy}{dx} = x^2 + y^2$ for $x = 0.4$ given that $y = 0$ when $x = 0$.
2. Use Picard's method to find $y(0.1)$ and $y(0.2)$,given that $\frac{dy}{dx} = x + y^2, y(0) = 0$.
3. Determine the value of y when $x = 0.1$ given that $y(0) = 1$and $y' = x^2 + y$.
4. Apply Runge-Kutta rule find the solution of the differential equation $y' = 3x + \frac{1}{2}y$ with $y_0 = 1$ at $x = 0.1$.

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II B.Sc Semester - IV

SUBJECT:MATHEMATICS

PAPER IV – REAL ANALYSIS

Question Bank

UNIT – I REAL SEQUENCE: 4 MARKS:

1.Prove that every convergent sequence is bounded.

2.Prove that limit of a convergence sequence is unique.

3.. If $s_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots \frac{1}{n+n}$ then show that $< s_n >$ is converges.

4.Show that lim $[\frac{(n+1)(n+2)......(n+n)}{n^n}] = \frac{4}{e}$

10 MARKS:

1. State and prove Sandwich theorem.

2. State and prove Monotone convergence theorem.

3.If $s_n = \frac{1}{1.2} + \frac{1}{2.3} + \cdots + \frac{1}{n(n+1)}$ then show that $< s_n >$ is converges.

4.If $s_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{n!}$ than show that $< s_n >$ is a convergent sequence.

5.Prove that every convergent sequence is Cauchy's sequence.

6. Prove that Cauchy's sequence is convergent sequence.

7.state and prove Cauchy's general principle of sequence.

UNIT II: INFINITE SERIES 4.MARKS :

Test the convergence of the following series.

1. $\sum \frac{\sqrt{n}}{n^2+1}$ 2. $\sum \frac{(n+1)(n+2)}{n^3\sqrt{n}}$ 3 $\sum \frac{1}{n^3} (\frac{n+1}{n+2})^n$ 4. $\sum \sqrt{n^2+1} - n$

5. $\sum \frac{(2n-1))}{n(n+1)(n+2)}$ 6. $\sum \frac{n^4}{n!}$ 7. $\sum \frac{n!}{n^n}$ 8. $\sum \frac{1.2.3.....n}{3.5.7....(2n-1)}$ 9. $\sum \frac{2^n}{n^n} n!$

10MARKS

1. State and prove limit comparison test.
2. Test the convergence of $\sum \sqrt{n^4+1} - \sqrt{n^4-1}$.
3. State and prove Cauchy's nth root test .
4. State and prove D'Alembert's ratio test .
5. Test the convergence of $\sum \sqrt{\frac{n}{n+1}}\, x^n$.
6. State and prove Leibnitz's test.

UNIT-III: LIMITS AND COTINUITY 4 MARKS:

1. Test the continuity of the function $f(x) = |x|$ for all x in R at the point x = 0 .

2. If $f(x) = x\, sin\frac{1}{x}\ for\ x \neq 0\ and\ f(0) = 0$ then find the continuity at x = 0.

3. Prove that every continuous function on [a, b] is uniformly continuous on [a,b].

10 MARKS

1.Prove that every continuous function is bounded on [a,b].

2. Prove that every continuous function on [a, b] is attains its bounds.

3 State and prove Intermediate value theorem .

4.If f(x) = $\frac{\sin(a+1)x + sinx}{x}$ for x < 0 , f(0)= c and f(x) = $\frac{(x+bx)^{\frac{1}{2}} - x^{1/2}}{bx^{3/2}}$ for x > o

then find the values of a, b and c.

UNIT IV: DIFFERENTIATION 4 MARKS:

1.Show that f(x) = |x|+|x-1| is not derivable at x=0,1.

2.Show that f(x) = x sin $\frac{1}{x}$ for x≠0 and f(0)=0 is not derivable at x=0.

3.Verify Rolle's theorem f(x) = $x^3 - 4x$ ∀ x∈[-2,2].

4. Verify Rolle's theorem f(x)=$(x - a)^m (x - b)^n$ for all x∈ [a , b].

5. Verify Rolle's theorem f(x) = log [$\frac{x^2 + ab}{x(a+b)}$] for all x∈ [a , b].

6.Verify Cauchy's mean value theorem $f(x) = x^2$, $g(x) = x^3$ for all $x \in [1, 2]$.

7 Verify Cauchy's mean value theorem $f(x) = \sqrt{x}$, $g(X) = \frac{1}{\sqrt{x}}$ for all $x \in [a, b]$.

8. Verify Cauchy's mean value theorem $f(x) = \frac{1}{x^2}$, $g(x) = \frac{1}{x}$ for all $x \in [a, b]$.

10 MARKS

1. State and prove Rolle's theorem.
2. State and prove Lagrange's mean value theorem .
3. State and prove Cauchy's mean value theorem.
4. Using Lagrange' s theorem show that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$ and hence deduce that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.
5. Using Lagrange' s theorem show that $\frac{\pi}{6} + \frac{\sqrt{3}}{15} < \sin^{-1} 0.6 < \frac{\pi}{6} + \frac{1}{8}$.
6. Verify Cauchy's mean value theorem $f(x) = e^x$, $g(x) = e^{-x}$ for all $x \in [a, b]$.

UNIT V: RIEMANN INTEGRATION 4 MARKS :

1. If $f(x) = k$ for all $x \in [a, b]$ where k is a real constant then show that f is integrable on [a, b] .

2. If $f(x) = 1$ when x is rational,

= -1 when x is irrational is not integrable on [a, b].

3 .If $f(x) = x^2$ for all $x \in [0, a]$ then show that $\int_0^a x^2 \, dx = \frac{a^3}{3}$.

4.If $f \in R[a, b]$ and m, M are the infimum and supremum of f on [a, b] then show that

$$m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$$

5. If $f \in R[a,b]$ then prove that $|f| \in R[a,b]$.

6. If $f \in R[a,b]$ then prove that $f^2 \in R[a,b]$.

10 MARKS.

1. State and prove Necessary and sufficient condition for Riemann integrable.
2. Prove that Every Continuous function on [a, b] is Riemann Integrable.
3. Prove that Every Monotonic function on [a, b] is Riemann-Integrable.
4. State and prove Fundamental theorem of Integral calculus.
5. State and prove First Mean value theorem of Riemann integral.
6. Show that $\frac{1}{\pi} \leq \int_0^1 \frac{\sin \pi x}{1+x^2}\, dx \leq \frac{2}{\pi}$.
7. Show that $\frac{\pi^3}{24} \leq \int_0^\pi \frac{x^2}{5+3\cos x}\, dx \leq \frac{\pi^3}{6}$

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III B.Sc, Semester - V

SUBJECT:MATHEMATICS

PAPER 7A –SPECIAL FUNCTIONS

Question Bank

UNIT -I : BETA AND GAMMA FUNCTIONS ,CHEBYSHEV POLYNOMIALS

10MARKS

1. **If n is +ve integer,prove that** $\Gamma\left(-n+\frac{1}{2}\right)=\frac{(-1)^n\ 2^n\ \sqrt{\pi}}{1.3.5......(2n-1)}$.
2. **Prove that** $\beta(l,m)=\frac{\Gamma(l)\,\Gamma(m)}{\Gamma(l+m)}$.
3. **Prove that** $\Gamma(m)\Gamma\left(m+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$.
4. **Show that** $\int_{-1}^{1}(1+x)^{p-1}\,(1-x)^{q-1}dx=2^{p+q-1}\frac{\Gamma(p)\,\Gamma(q)}{\Gamma(p+q)}$.
5. **Prove that** $\int_{-1}^{1}\frac{x^2}{(1-x^4)^{\frac{1}{2}}}\,dx\ \times\int_{0}^{1}\frac{dx}{(1+x^4)^{\frac{1}{2}}}\,dx==\frac{\pi}{4\sqrt{2}}$.
6. **Show that** $2^n\ \ \Gamma\left(n+\frac{1}{2}\right)=1.3.5\ldots.(2n-1)\sqrt{\pi}$ where n is positive integer.

4 MARKS:

1. **Show that** $\Gamma\left(\frac{3}{2}-x\right)\Gamma\left(\frac{3}{2}+x\right)=\left(\frac{1}{4}-x^2\right)$.
2. **Evaluate** $\int_{0}^{\propto}\frac{x^4\ (1+x^5)}{(1+x)^{15}}dx$.
3. **Evaluate** $\int_{0}^{\propto}\frac{x^8\ (1-x^6)}{(1+x)^{24}}\ dx$.
4. **Evaluate** $\int_{0}^{2}x(8-x^3)^{\frac{1}{3}}dx$.
5. **Show that** $\Gamma\left(\frac{1}{2}+x\right)\Gamma\left(\frac{1}{2}-x\right)=\frac{\pi}{cos\pi x}$.

6. **Evaluate** (i) $\int_0^1 x^4 \quad (1-x)^2 dx.$ (ii) $\int_0^\infty x^4 \quad e^{-x} dx.$

7. Evaluate $\Gamma\left(-\frac{1}{2}\right), \Gamma\left(-\frac{3}{2}\right), \left(-\frac{5}{2}\right)$.

UNIT -II : POWER SERIES

10 MARKS

1. Find the radius of Convergence and the exact intervel of Convergence of $\Sigma \frac{(n+1)}{(n+2)(n+3)} x^n$.

2. S·T X=0 is an irregular Singular Point and x=-1 is regular singular point of $x^2 (x+1)^2 y^{II} + (x^2-1)y^I + 2y = 0$

3. Show that X=0 is a regular Singular Point of $x^2y^{II} + xy^I + (x^2 - \frac{1}{4})y = 0$.

4. Find the Power Series solution of the equation $(x^2-1) y^{II} + x y^I - y = 0$.

4 MARKS

1. Find the radius of Convergence of $\Sigma \frac{(n+1)}{n(n+2)} x^n$.
2. Find the radius of Convergence of $\Sigma \frac{(2n!)}{(n!)2} x^{2n.}$
3. Show that x=0 is an ordinary point of $y^{II} - x y^I + 2y = 0$.
4. Show that x=0 is an ordinary point of $(x^2+1) y^{II} + x y^I - xy = 0$.

UNIT -III : HERMITE POLYNOMIALS

10 Marks:

1. Prove that $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}\left(e^{-x^2}\right)$.
2. Prove that $e^{2tx-t^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x)$.
3. Show that $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$.
4. Prove that $\int_{-\infty}^{+\infty} e^{-x^2} H_n(x)\, H_m(x) dx = \begin{cases} 0, \ if \ m \neq n \\ \sqrt{\pi}\ 2^n\ n!\, if \ m = n \end{cases}$

4 Marks :

5. **Show that** $H_n'(x) = 2nH_{n-1}(x)\ if\ n \geq 1$. (Recurrence I)
6. **Show that** $H_n(x) = 2^n \left[e^{(\frac{-d^2}{4dx^2})x^n} \right]$.
7. **Find polynomials** $H_0(x), H_1(x), H_2(x)$.
8. **Show that** $H_n'(x) = 2xH_n(x) - H_{n+1}(x)$
9. **Show that** $H''_n(x) - 2xH'_n(x) + 2nH_n(x) = 0$
10. **Show that** $H''_n(x) = 4n(n-1)H_{n-2}(x)$

UNIT -IV : LEGENDRE POLYNOMIALS

10 Marks:

1.Show that ($(1\text{-}2xh\text{+}h^2) = \sum_{n=0}^{\infty} h^n\, p_{n(x)}$.

2.Prove that $\int_{-1}^{1} [P_{n(X)}\ ]\, dx = \frac{2}{2n+1}\ if\ \ m = n$

3.Prove that $(2n+1)xp_n = (n+1)p_{n+1\ +n}\ p_{n-1}$

4. Prove that $P_n(x) = \frac{1}{\pi}\int_0^{\pi} [\ x \pm \sqrt{x^2}\text{-}1\ \cos\varphi]^n\ d\varphi$

5. Prove that $P_n(x) = \frac{1}{\pi}\int_0^{\pi} \frac{d\varphi}{[x \pm \cos\varphi]n+1}$

4 Marks :

6. Prove that $P_n(x) = x\, P_n' \text{ - } P_{n-1}'$
7. Prove that $(2n+1)\, xP_n(x) = (n+1)\, P_{n+1}' + nP_{n-1}'$
8. Prove that $(n+1)\, P_n(x) = P_{n+1}' \text{ - } xP_n'$
9. Prove that $(1 - x^2)\, P_n' = n(p_{n-1} - xp_n)$
10. prove that $(1 - x^2)\, P_n' = (n+1)(xp_{n\ -}\ p_{n+1})$

UNIT V : BESSELS EQUATIONS

10 Marks:

1.Prove that x $J_n'(x)$= n J_n(x)-x J_{n+1} (x) .

2.Prove that 2 $J_n'(x)$= J_{n-1}(x)- J_{n+1} (x) .

3.Prove that 2 n J_n(x)=x[J_{n-1}(x)+J_{n+1} (x)].

4.Prove that $e^{\frac{x(Z-1/2)}{2}}$= $\sum_{n=-\infty}^{\infty} Z^n$ J_n(x).

5. Show that i) $\sqrt{\frac{\pi X}{2}}$ $J_{\frac{3}{2}}$ (x) = $\frac{1}{X}$ sinx-cosx ii) $J_{\frac{5}{2}}$ = $\sqrt{\frac{2}{\pi x}}$ [$\frac{3-x^2}{x^2}$ $sinx - \frac{3}{x}$ $cosx$]

6. Prove that J_{-n}(x)=$(-1)^n J_n$(x)

4 Marks :

1) Show that i) $J_{\frac{-1}{2}}(x)$= $\sqrt{\frac{2}{\pi x}}$ cosx ii) $J_{\frac{1}{2}}(x)$= $\sqrt{\frac{2}{\pi x}}$ sinx
2) Prove that i) J_0'= $-J_1$
   ii) J_2 = J_0'' - $x^{-1}J_0'$
   iii) J_2 - J_0 = 2 J_0''
3) J_n(-x) = $(-1)^n$ J_n(x)
4) prove that $\frac{d}{dx}$[x J_n(x) J_{n+1} (x)] = x[$J_n^2(x) - J_{n+1}^2(x)$]
5) prove that $\frac{d}{dx}$[$J_n^2(x)$] = $\frac{x}{2n}$ [$J_{n-1}^2(x) - J_{n+1}^2(x)$]
6) prove that J_n''(x) = $\frac{1}{4}$ [J_{n-2}(x)- 2J_n (x) + J_{n+2} (x)]

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I B.Sc Semester - II
SUBJECT:MATHEMATICS
PAPER II – SOLID GEOMETRY
Question Bank

UNIT-I:The Plane   10M (2) +4M (2)

4 marks:

1. Find the equation of a plane through the points (2,2,1) (9,3,6) and perpendicular to a plane 2x+6y+6z=9
2. Find the equation of a plane through the point (-1,6,2) and perpendicular to the planes x+2y+2z=5,3x+3y+2z-8=0
3. Show that the points (-6,3,2),(3,-2,4)(5,7,3),(-13,17,-1) are coplanar.
4. If a plane meets the co-ordinate axes in A,B,C such that the centroid of ΔABC is the point (p, q, r) then show that the equation of the plane is $\frac{x}{p}+\frac{y}{q}+\frac{z}{r}=3$.
5. If the plane 2x+3y+kz-7=0 is perpendicular to a plane 2x-y+3z=9 find k
6. Find the equation of the plane through the point (4,0,1) and pallel the the plane 4x+3y-12z+6=0

10 marks:

1. A variable plane is at a constant distance 3p from the origin and meets the axes in A,B,C. show that the Locus of centroid of ΔABC is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$
2. A variable plane is at a constant distance p from the origin and meets the axes in A,B,C. show that the Locus of centroid of the teteahedron OABC is $x^{-2} + y^{-2} + z^{-2} = 16\,p^{-2}$
3. Find the equation of the bisectors of the angle between the planes 3x - 6y + 2z + 5=0, 4x - 12y + 3z - 3=0 and which contains the origin

4. Find the equation of the bisectors of the angles between the planes $2x-y+2z+3=0, 3x-2y+6z+8=0$ and distinguish them
5. Show that $x^2+4y^2+9z^2-12yz-6zx+4xy+5x+10y-15z+6=0$ represents a pair of parallel planes and find the distance between them
6. Show that the equation $2x^2-6y^2-12z^2+18yz+2zx+xy=0$ represents pair of planes and find the angle between them.
7. Show that the equation $x^2+4y^2+4z^2+4xy+8yz+4zx-9x-18y-18z+18=0$ Represent a pair of parallel palnesn hence find the distance between them.

UNIT-II:The Right Line 10M (2) +4M (2)

4 marks:

1. Find the image of the point (1,-1, 5) in the plane $3x - 2y - 4z - 14 = 0$.
2. Find the symmetrical form of the line $2x+2y-z-6 = 0 = 2x+3y-z-6$
3. Find the image of (1,6,3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.
4. Show that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}, \frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar find the plane containing them.
5. Show that $\frac{x+1}{1}=\frac{y+1}{2}=\frac{z+1}{3}$ and $x+2y+3z-8 = 0 = 2x+3y+4z-11$ are intersect. Find the point of intersection.
6. Find the condition the lines $x = az + b, y=cz + d$ and $x = a_1z + b_1, y=c_1z + d_1$ are perpendicular.
7. Find the angle between the lines $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $3x+y+z=7$.

10 marks:

1. Find the image of the line $\frac{x-1}{2}=\frac{y}{-1}=\frac{z+2}{2}$ in the plane $2x+y-3z-4=0$.
2. Show that the lines $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}, \frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ are coplanar, find their point of intersection and the plane containing them.
3. Show that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}, \frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar, find their point of intersection and the plane containing the lines.

4. Find the S.D and equation between the lines . $\frac{x}{2}=\frac{y}{-3}=\frac{z}{1}$, $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$.

5. Find the S.D and equation between the lines $\frac{x-1}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$, $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$..

   Also find the points in which the S.D line meets the given lines.

6. Find the length and equation of S.D line between the lines $\frac{x}{4}=\frac{y+1}{3}=\frac{z-2}{2}$,

   5x-2y-3z+6=0=x-3y+2z-3.

7. Show that the equation to the plane containing the line $\frac{y}{b}+\frac{z}{c}=1$, x=0 and parallel to the line $\frac{x}{a}-\frac{z}{c}=1$, y=0 is $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}+1=0$ and if 2d is the S.D prove that $\frac{1}{d2}=\frac{1}{a2}+\frac{1}{b2}+\frac{1}{c2}$

UNIT-III: The Sphere 10M (2) +4M (2)

4 marks:

1. A plane passes through a fixed point (a, b, c) and cuts the axes in A,B,C. Show that the locus of the centre of the sphere OABC is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$.
2. A sphere of constant radius **k** passes through the origin and meets the axes in A,B,C. Show that the locus centroid of ΔABC is the sphere $9(x^2+y^2+z^2)=4k^2$.
3. A plane passes through a fixed point (a, b ,c).show that the foot of the perpendicular to the plane from the origin lies on the sphere $x^2+y^2+z^2-ax-by-cz=0$.
4. Find the centre and radius of the circle $x^2+y^2+z^2-2y-4z=11$, x+2y+2z=15.
5. Find the equation of a sphere through the circle $x^2+y^2+z^2+7y-2z+2=0$,2x+3y+4z=8 is a great circle.
6. Show that the plane 2x-2y+2z+12=0 touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the point of contact.
7. Find the equation of sphere which touches the plane 3x+2y-z+2=0 at (1,-2,1) and passes through the origin.

8. Find the equation of tangent planes to the sphere $x^2+y^2+z^2-4x+2y-6z+5=0$ which are parallel to $2x+2y-z=0$.
9. show that spheres $x^2+y^2+z^2-2x-4y-6z-50=0, x^2+y^2+z^2-10x+2y+18z+8z=0$ touch externally at $(\frac{45}{13},\frac{2}{13},\frac{-57}{13})$

10 marks:

1. Find the equation of a sphere through the points (1,-3,4),(1,-5,2),(1,-3,0) and whose centre lies on $x + y + z = 0$.
2. Find the equation of a sphere through the points (1,0,0)(0,1,0)(0,0,1) and having the least radius.
3. Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and find the point of contact .
4. Show that the circles $x^2+y^2+z^2-y+2z=0$, $x - y + z=0$, $x^2+y^2+z^2+x-3y+z-5=0$, $2x-y+4z-1=0$ lie on the same sphere. Also find its equation.
5. Find the equation of the spheres passes through the circle $x^2+y^2+z^2=4$, $z=0$ and intersected by the plane $x+2y+2z=0$ in the circle of radius 3.
6. Find the equation of a sphere which touches the plane $3x+2y-z+2=0$ at (1,-2,1) and cut orthogonally the sphere $x^2+y^2+z^2-4x+6y+4=0$.
7. Find the equation of the sphere which touches the plane $3x+2y-z+2=0$ at (1,-2,1) and passes through the origin .

UNIT-IV:The Sphere and cones 10M (2) +4M (1)

4 marks:

1. Find the pole of the plane $x+2y+3z=7$
2. Find the plane of contact of the point (3,-1,5) w.r.t the sphere , $x^2+y^2+z^2-2x+4y+6z-11=0$.
3. Show that the spheres , $x^2+y^2+z^2+6y+2z+8=0$, $x^2+y^2+z^2+6x+8y+4z+20=0$ are orthogonal .

4. Find the equation of the cone with vertex at (-1,1,2)guiding curve $3x^2 - y^2 = 1, z = 0$

5. Find the equation of the lines of intersection of yhe plane and the cone given below $x+3y-2z=0$, $x^2+9y^2-4z^2=0$.

6 .Find the equation of the cone whose vertex is (1,2,3) and base $y^2=4ax, z=0$.

10 marks:

1. If r_1, r_2 are radii of two orthogonal spheres then prove that the radius of their intersection $r_1 r_2 / \sqrt{(r_1^2+r_2^2)}$.
2. Find the radical centre of the spheres
$x^2+y^2+z^2+4y=0$, $x^2+y^2+z^2+2x+2y+2z+2=0$,
$x^2+y^2+z^2+3x-2y+8z+6=0$, $x^2+y^2+z^2-x+4y-6z-2=0$
3. Find the limiting points of the coaxial system $x^2+y^2+z^2+20x+30y-40z+29+\lambda(12x-3y+4z)=0$.
4. Find the limiting points of the coaxial system of spheres
$x^2+y^2+z^2+3x-3y+6=0, x^2+y^2+z^2-6y-6z+6=0$.
5. Find the equation of a sphere which touches the plane $3x+2y-z+2=0$ at (1,-2,1) and cut orthogonally the sphere $x^2+y^2+z^2-4x+6y+4=0$.
6. Find the equation of the quadric cone through the co-ordinate axis the three lines $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}, \frac{x}{-1} = \frac{y}{1} = \frac{z}{1}$ & $\frac{x}{5} = \frac{y}{4} = \frac{z}{1}$.
7. Find the angle between the lines of intersection of the plane $x-3y+z=0$ and the cone $x^2-5y^2+z^2=0$.
8. Prove the angle between the lines of intersection of the plane $x+y+z=0$ With the cone $ayz+bzx+cxy=0$ is $\frac{\pi}{3}$ if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

UNIT-V: The cone 10M (2) +4M (1)

4 marks:

1. Find the equation of the cone with vertex at (-1,1,2)guiding curve

$$3x^2 - y^2 = 1, z = 0$$

2.find the enveloping cone of the sphere $x^2+y^2+z^2+2x-2y=2$ with its vertex at(1,1,1)

3.Find the equation to the right circular cone whose vertex is (3,2,1), axis $\frac{x-3}{4}=\frac{y-2}{1}=\frac{z-1}{3}$ and semi vertical angle 30^0

4. Show that the reciprocial cone of $ax^2+by^2+cz^2=0$ is the cone $\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 0$

10 marks:

1. Find the vertices of the cone $2x^2+2y^2+7z^2-10yz-10zx+2x+2y+26z-17=0$
2. Find the vertices of the cone $7x^2+2y^2+2z^2-10zx-10xy+26x-2y+2z-17=0$
3. Prove that the equation $\sqrt{fx} \pm \sqrt{gy} \pm \sqrt{hz} = 0$ represents a cone that touches the co-ordinate planes and find its reciprocal cone .
4. Show that the general equation to a cone which touches the three cordinate planes is $\sqrt{ax} + \sqrt{by} + \sqrt{cz} = 0$
5. Show that a right circular cone has sets of three mutually perpendicular generators, its semi verticle angle must be $\tan^{-1}(\sqrt{2})$
6. Show that the semi verticle angle of a right circular cone having three mutually perpendicular i)generators is $\tan^{-1}(\sqrt{2})$ ii) tangent planes is $\tan^{-1}(\frac{1}{\sqrt{2}})$